M.Sc. Second Year (Mathematics) (New CBCS Pattern) Semester-III

PSCMTH12 - Functional Analysis

P. Pages : 2

GUG/S/25/13756

Time : Three Hours

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Max. Marks : 100

Notes : 1. All **five** questions are compulsory.
2. Each question carries equal marks.

UNIT – I

1. a) Let M be a closed linear subspace of a normed linear space N. If the norm of a coset x + M in the quotient space N/M is defined by **10**

$\|x+M\| = \inf\{\|x+m\|; m \in M\}$,

then prove that N/M is a normed linear space. Further, if N is a Banach space then prove that so is N/M.

b) State and prove The Hahn – Banach Theorem. **10**

OR

c) If N is Normed linear space, then prove that The closed unit sphere S* in N* is a compact Hausdorff space in the weak* topology. **10**

d) If N is Normed linear space and x_0 is a non-zero vector in N, then prove that there exist a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$. **10**

UNIT – II

2. a) If B and B' are Banach spaces and if T is a continuous linear transformation of B onto B', then prove that the T is an open. **10**

b) Prove that A non-empty subset X of a normed linear space N is bounded $\Leftrightarrow f(x)$ is a bounded set of Numbers for each f in N^*. **10**

OR

c) Prove that A closed convex subset C of a Hilbert space H contains a unique vector of smallest Norm. **10**

d) If $\{e_i\}$ is an orthonormal set in a Hilbert space H. Then prove that $\Sigma |(x_i, e_i)|^2 \le \|x\|^2$ for every point x in H. **10**

UNIT – III

3. a) Let H be a Hilbert space, and let f be an arbitrary functional in H*. Then prove that there exist a unique vector y in H such that **10**

$$f(x) = (x,y)$$

for every x in H.

b) If T is an operator on H for which $(Tx, x) = 0$ for all x, then prove that T = 0. **10**

OR

c) If P is a projection on N, with range M and Null space N then prove that $M \perp N \Leftrightarrow P$ is self adjoint and in this case $N = M^{\perp}$. **10**

d) If T is an operator on H, Then prove that T is Normal If and only if its Real part and imaginary part commute. **10**

UNIT – IV

4. a) Prove that two matrices in A_n are similar $\Leftrightarrow$ they are matrices of a single operator on N relative to different bases. **10**

b) If T is Normal, then prove that $M_i's$ are pairwise orthogonal. **10**

OR

c) If T is Normal, then prove that $M_i'S$ span H. **10**

d) Let B be a Basis for H, and T an operator who be matrix relative to B is $[\alpha ij]$. Then prove that T is non-singular $\Leftrightarrow [\alpha ij]$ is Non-singular, and in this case $[\alpha ij]^{-1} = [T^{-1}]$ **10**

5. a) Define the Natural imbedding of N in N**. **5**

b) If $\langle e_i \rangle$ is an orthonormal set in a Hilbert space H_1 and If x is any vector in H_1 then prove that the set $s = \{e_i = (x, e_i) \neq 0\}$ is either empty or countable. **5**

c) If A_1 and A_2 are self-adjoint operators on H, then prove that their product A_1, A_2 is self - adjoint $\Leftrightarrow A_1 A_2 = A_2 A_1$. **5**

d) If T is Normal, then prove that each M_i reduces T. **5**

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